Abstract—During the last decade, Support Vector Machines (SVM) have proved to be very successful tools for classification and regression problems. The representational performance of this type of networks is studied on a cavity flow facility developed to investigate the characteristics of aerodynamic flows at various Mach numbers. Several test conditions have been experimented to collect a set of data, which is in the form of pressure readings from particular points in the test section. The goal is to develop a SVM based model that emulates the one step ahead behavior of the flow measurement at the cavity floor. The SVM based model is built for a very limited amount of training data and the model is tested for an extended set of test conditions. A relative error is defined to measure the reconstruction performance, and the peak value of the FFT magnitude of the error is measured. The results indicate that the SVM based model is capable of matching the experimental data satisfactorily over the conditions that are close to the training data collection conditions, and the performance degrades as the Mach number gets away from the conditions considered during training.

I. INTRODUCTION

Feedback control performance on aerodynamic flow systems heavily depends upon the capabilities of a representative model. The process under investigation is highly nonlinear since the governing dynamics is described by Navier-Stokes equations, which display quite complicated behavior in aerodynamic flows and this entails high sampling rates. The described nature of the problem highlights how substantial the performance at the modeling stage is. Modeling of such a process using machine learning methods is one alternative that is motivated also by the facts that real-time observations are generally noise corrupted and even rough models of the system dynamics are unavailable. From an input-output data processing point of view, the problem in hand is a good test bed where machine learning algorithms can be applied. This paper focuses on an increasingly popular tool named Support Vector Machines.

In the past years, several soft computing tools have been used for modeling of aerodynamic systems, for example Neural Networks (NN) in [1-7], fuzzy logic in [8-11] and SVM in [12].

In [1], Jacobson and Reynolds conducted a numerical study on the control of wall shear stress in a boundary layer by using feedforward NN as inverse controllers, which showed skin friction reduction by about 8%. The study of active laminar flow control [2] showed that a properly trained NN can establish complex nonlinear relationships between multiple inputs and outputs which are peculiar to an active flow control system. The work demonstrates the cancellation of wave disturbances in transitional boundary layers by a pretrained neural models. Sensors measure either wall pressure or wall shear stress. Training strategies and performance measures are considered, and fault tolerance capability of NN is emphasized. Faller et al., [3], obtained a NN model of a pitching airfoil based on experimental data. With limited training data, the model predicts unsteady surface pressure topologies within 5% of what is available in the experimental data. Given the actuator control signals, the NN anticipates the interactions between the unsteady flow field and airfoil. The NN has a very complex structure configuration. Gradient descent is used for training and the pressure values on the airfoil are estimated by using the recordings of angle of attack and its time derivative. The NN controller has 6-12-12-1 configuration, and a desired lift/drag ratio is aimed to be observed. It is possible to extend the results focusing on NN use in aerodynamic system modeling (See [4-7] and the references therein).

As another alternative, Fuzzy Logic (FL) is a practical framework for solving complicated problems by utilizing expert knowledge. The practicality of the paradigm stems from the fact that the human expertise is expressed in the form of IF antecedent THEN consequent statements, i.e. the task to be achieved is modeled through the use of linguistic descriptions. In [8], Cohen et al. use FL for the control of a circular cylinder vortex shedding model. The fuzzy system in [8] has been used to scale a control signal produced by a PID controller, and it has been shown that such a strategy yields significant improvement in the performance compared to the sole PID solution. In [9], FL with triangular membership functions is used for controlling the vortex flows on a generic X29-A model. The fuzzy controller is compared with neural
controllers and predictive schemes. Dragojlovic et al. utilize the fuzzy logic in improving the performance of a Computational Fluid Dynamics (CFD) solver. The fuzzy control scheme guides the increment in the relaxation factor by using triangular membership functions, [10-11]. Depending on the past solution entries, the CFD solver automatically adjusts itself to exploit the best relaxation factor.

In 1995, Vapnik proposed a new approach for classification and regression problems, named support vector machines, [13]. This new approach aims at minimizing the structural risk, i.e. the upper bound of the generalization error. By this means, SVMs are superior to conventional NN, the training algorithms of which minimize the empirical risk over a set of training pairs, [14-15]. In [12], five hole pressure probe calibration is studied comparatively with NN models and it is seen that SVM predictions are much better than those obtained with NN trained under the same operating conditions. In the same paper, it is shown that utilizing the SVMs, efficiency of the response surface technique can be increased for CFD based shape optimizations. As the test bed, diffusers converting the the dynamic pressure to static pressure rise is chosen and response surface has been constructed with the aid of SVM. In [16], Gretton et al., present a SVM based identification of a robot arm and the regressor used in [16] has a similar structure as we use in this paper.

As outlined above, some work has been done in the past decade to explore the use of machine learning techniques in flow modeling and control with various degrees of success. Several of these works showed promising results but were based on numerical simulations and lacked any experimental validation of the concept. The few experimental studies available are concerned with slowly varying states of the flow. To the best of our knowledge, no attempt has been made so far in using SVM to model a more dynamic, higher frequency flow like the one over a cavity. Therefore, many questions remain open about the merit and effectiveness of the tools exploiting statistical learning theory in flow modeling and control. Having this motivation in mind, in this paper, we work on the experimental setup shown in Figs. 1-2 and introduced in the next section. The goal is to characterize the flow passing over a cavity based on surface pressure measurements. The third section summarizes the modeling based on SVM. The obtained simulation results are discussed in the fourth section, and conclusions constitute the last part of the paper.

II. THE EXPERIMENTAL FACILITY

In this study, the experimental facility described in more detail in [17-18] is used. The core of the experimental setup consists of an optically accessible, blow-down type wind tunnel with a test section of equal width and height, \( W = H = 50.8 \text{ mm} \). A cavity that spans the entire width of the test section is recessed in the floor with a depth \( D = 12.7 \text{ mm} \) and length \( L = 50.8 \text{ mm} \) for an aspect ratio \( L/D = 4 \). For control, the cavity shear-layer is forced by a 2-D synthetic-jet type actuator issuing from the end slot of a high-aspect-ratio converging nozzle embedded in the cavity leading edge and spanning the width of the cavity, see Fig. 2. Actuation is provided by the movement of the titanium diaphragm of a Selenium D3300Ti compression driver whose input signal is amplified by a Crown D-150A amplifier. The pressure fluctuations are measured by Kulite dynamic pressure transducers placed in different locations in the test section, see Fig. 3.

Since the experimental facility enables us to acquire pointwise observations from the critical locations of the cavity, one could use this information for estimation of the flow inside the cavity. This is done using a dSPACE 1103 DSP board connected to a Dell Precision Workstation 650. This system acquires the pressure transducer signals simultaneously at a sampling frequency of 50 kHz through 16-bit input channels, and manipulates them to produce the desired output signal from a 14-bit output channel. Each recording is band-pass filtered between 200Hz and 10 kHz to remove spurious frequency components. The simultaneous time traces collected from these transducers have been used to train the support vector network with the characteristics described in [19-21]. It is critically important to emphasize that the data must be spectrally rich enough to capture cases that are likely to be encountered in real-time operation. This makes sure that the NN responds appropriately to the input variables.

In [18], it is observed that the cavity flow exhibits strong, single-mode resonance in the Mach number ranges 0.25-0.31 and 0.39-0.5, and multi-mode resonance in the Mach number range 0.32-0.38. In the same study, it is observed that the frequency of sinusoidal forcing with the synthetic jet-like actuator has a major impact on the cavity flow resonance whereas the effect of the amplitude is relatively minor and it affects the control authority only at higher Mach numbers. This prompted the development of a logic-based type of control that searches the forcing frequencies in a closed-loop fashion that reduce the cavity flow resonant peaks and then maintains the system in such conditions through an open-loop control. The technique performed well in the experimental trials and allowed identification of optimal frequencies for the reduction of resonant peaks in the Mach number range 0.25-0.5. Another indication of this result was the adequate control authority introduced by the actuators. Some effort within the described research has been dedicated to design classical controllers and these succeeded to some extent. The experience gained during these trials have stipulated that the modeling of the process deserves particular attention as the desired closed loop control performance depends strictly upon the representational capability of the process model. Since the experimental facility enables us to acquire pointwise observations from the physically critical locations of the cavity, one could use this information for identification of the cavity flow and this paper discusses how SVMs could be utilized for this purpose.

III. MODELING BY SUPPORT VECTOR MACHINES

A. SVM Basics

Consider the regression problem over the pairs
\[ D = \{(u_1, d_1), \ldots, (u_N, d_N)\}, \quad u_i \in \mathbb{R}^m, \quad d \in \mathbb{R} \] (1) with a function
\[ f(u) = (w, u) + b \] (2)
where \( w \) and \( b \) denote the weight vector and the bias value, respectively. \( \langle \cdot, \cdot \rangle \) stands for an appropriately defined operator, which is an inner product for linear regression and a kernel for nonlinear regression. Defining a quadratic loss function as in (3) quantifies the performance for the \( i \)th pair,
\[ L(d_i, f(u_i)) = (d_i - f(u_i))^2. \] (3)
Minimizing the empirical risk given by (4) lets us obtain the best values of \( w \)s causing least complexity represented by \( ||w||^2 \);
\[ R = \frac{1}{2} ||w||^2 + C \sum_{i=1}^{N} L(d_i, f(u_i))^2, \] (4)
where \( C \) is a parameter determining the relative importance of the terms contributing to \( R \). The primal form of the optimization problem can be expressed compactly as
\[ \min_{w, b} \frac{1}{2} ||w||^2 + C \sum_{i=1}^{N} (\zeta_i^2 + \hat{\zeta}_i^2) \] (5)
\[ \text{s.t.} \begin{cases} f(u_j) - d_j \leq \zeta_j, \\ d_j - f(u_j) \leq \hat{\zeta}_j, \quad j = 1, 2, \ldots, N \end{cases} \] (6)
where \( \zeta_j \) and \( \hat{\zeta}_j \) are slack variables penalizing the deviations from the target output. The above described problem can be converted into a convex quadratic optimization problem by writing the dual representation. The solution can be obtained by introducing the Lagrange multipliers and performing the following minimization for \( \beta \in \mathbb{R}^N \);
\[ \min_{\beta} \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \beta_i \beta_j \langle u_i, u_j \rangle - \sum_{i=1}^{N} \beta_i d_i + \frac{1}{2C} \sum_{i=1}^{N} \beta_i^2, \] (7)
with constraint \( \sum_{i=1}^{N} \beta_i = 0 \). It should be noted that the support vectors are the \( u_i \)s for which the corresponding \( \beta_i \) is nonzero. The result of the minimization lets us obtain
\[ w^* = \sum_{i=1}^{N} \beta_i u_i, \] (8)
\[ b^* = \frac{1}{N} \sum_{i=1}^{N} \left( y_i - \sum_{j=1}^{N} \langle u_i, u_j \rangle \beta_j \right) \] (9)
which are to be used in (2). The nonlinear regression problem is to replace the operator \( \langle \cdot, \cdot \rangle \) in (1) with a kernel function satisfying the Mercer conditions, [15].
B. Application of SVM for Aerodynamic Flow Modeling

In the training phase, the SVM based model is asked to realize the mapping from current state of the flow and external excitation to the next state of the flow. The state of the flow is described by the information acquired from the chosen sensors. According to Figs. 2-3, the sensor labelled $S_1$ measures $u_{1,k}$, the actuation signal at time $k$ in Volts, $S_2$ measures $u_{2,k}$, the pressure fluctuations just before the actuator exit, $S_3$ measures $u_{3,k}$, the pressure fluctuation just after the actuator exit (i.e. at the shear layer receptivity region just downstream of the cavity leading edge), $S_4$ measures $u_{4,k}$, the pressure fluctuations (if any) before the cavity, $S_5$ measures $u_{5,k}$, the pressure fluctuations at the cavity trailing edge, $S_6$ measures $d_k$, the pressure fluctuations at the center of the cavity floor. The signals from these transducers are simultaneously sampled with the host computer.

With these definitions, a series-parallel SVM based emulator is desired to match the training data in (1). It should be noted that the input vector is composed of the information coming from the above sensors and their delayed (past) values, whose delay depths are specified by the designer. Notice that the Mach number could also be an external input to the SVM model to characterize the dynamical composition of various experimental regimes within a single support vector network. If such an approach succeeds, we obtain a SVM emulator that can be used at Mach numbers around Mach = 0.30 regime. Towards this goal, we have collected a set of experimental data for several test cases as tabulated in Table I.

<table>
<thead>
<tr>
<th>Mach Number</th>
<th>Excitation Frequency</th>
<th>Excitation Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.28</td>
<td>3250Hz</td>
<td>2.35V rms</td>
</tr>
<tr>
<td>0.28</td>
<td>3920Hz</td>
<td>4.06V rms</td>
</tr>
<tr>
<td>0.32</td>
<td>3250Hz</td>
<td>2.35V rms</td>
</tr>
<tr>
<td>0.32</td>
<td>3920Hz</td>
<td>4.00V rms</td>
</tr>
</tbody>
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Every experiment shown above contributes only 126 samples to the training data set, which excludes Mach = 0.30 case. This is a deliberate choice for test data as Mach = 0.30 displays quite rich spectral view making the corresponding phenomenon difficult to model compactly. The total number of training samples is 504, which provides clearly very limited information to perform a satisfactory modeling. One might prefer to enlarge the training data set to cover a richer set of cases yet the cost of this is a significant increase in the training time.

At discrete time index $k$, the input vector to the SVM is as given below

$$u_k = (u_{1,k}, u_{0,k}, u_{6,k-1}, u_{4,k}, u_{5,k}, \text{Mach}).$$

The desired output for this input pattern is $d_k = u_{6,k+1}$.

In order to validate the modeling claim of the paper, the mechanism in (1) is implemented with the discussed SVM structure having 6 inputs, and one output. The training has been achieved by using the software available at http://www.isis.ecs.soton.ac.uk. The rationale that lies behind is the minimization of the discrepancy between the process outputs and the SVM model response over a set of input-output pairs while maintaining the minimal structural risk. A linear kernel is utilized, i.e. kernelized value of $u_k$ and $u_l$ is $\langle u_k, u_l \rangle = u_k u_l^T$ and the SVM model is obtained approximately after a 2.3 hours of training process.

In Fig. 4, the validation of the obtained SVM model is shown for one of the unseen operating conditions (in terms of Mach number), which correspond to the case described by Mach = 0.30. In this figure, $d_k$ and $x_k$ denote the desired (already recorded) value and prediction of SVM based model $x_{k+1} = f(u_k)$, respectively. The obtained results are reasonably good to claim that the model functions well for the considered operating conditions. We can quantify this by defining the relative error $e_{rel}$ as the ratio of the average powers of $d$ and $d - x$ over the time interval $t \in [0, T_f]$, where $T_f = 163.84 \text{msec.}$, that is

$$e_{rel} := \frac{1}{T_f} \int_0^{T_f} |d(t) - x(t)|^2 dt.$$

The numerical results presented in Fig. 4, give $e_{rel} = 0.0431$, i.e. average power of the error signal $d(t) - x(t)$ is 4.31% of the average power of the signal $d(t)$. Clearly from (11), the smaller the $e_{rel}$ the better the reconstruction performance.

To sum up, when looking at the result illustrated in Fig. 4, the similarity of the desired and estimated signals is found to be promising.

Although the similarity in time domain is one way of demonstrating the performance we need to check the spectral views to strengthen the theoretical claims. In Fig. 5, we demonstrate the Fast Fourier Transform (FFT) of the signals involved in the procedure. The upper subplot depicts the FFT magnitudes of $d(t)$ and $x(t)$ over the 200Hz-10kHz band of the spectrum. Obtaining a similarity over this range of frequencies is sufficient as the important information is present in this band. The lower subplot illustrates the FFT magnitude of the difference $d(t) - x(t)$. The resonant peak is visible in both subplots and the peak value in the lower subplot, which is reasonably small, emphasizes that the phase of the prediction reasonably fits the desired signal.

In Fig. 6, we zoom the behavior in the vicinity of the peak at 3920Hz. The two FFT magnitude plots are very close to each other, which stipulate that the SVM based model performs well under the depicted operating conditions.

In Table II, we summarize the results for an extended set of operating conditions including the one above. In each case, we compute the relative error in (11) and the peak value of the FFT magnitude of the error $d(t) - x(t)$. We consider Mach numbers 0.25, 0.28, 0.30, 0.32 and 0.35. For each of these cases, we perform three sets of experiments. First experiment set comprises the noise driven cases, i.e. the actuator is excited by a noise signal within the allowed physical limits of the actuator. The second set of experiments provides data for the excitation by a sinusoidal signal having frequency $f_{exc} = 3250$Hz and amplitude $A_{exc} = 2.35$V rms.

The cases in the third experiment set is similar to those in
Fig. 4. The time view of the signals and the error for Mach 0.3. The excitation signal is a sinusoidal signal having frequency 3920Hz and magnitude 4.06Vrms.

Fig. 5. Spectral view of the signals and the error for Mach 0.3. The excitation signal is a sinusoidal signal having frequency 3920Hz and magnitude 4.06Vrms.

Fig. 6. Zoomed spectral view of the signals and the error for Mach 0.3. The excitation signal is a sinusoidal signal having frequency 3920Hz and magnitude 4.06Vrms.

TABLE II

| Mach | \(f_{\text{exc}}\) | \(A_{\text{exc}}\) | \(r_{\text{rel}}\) | sup \(|\text{FFT}(d - x)|\) |
|-------|-----------------|-----------------|-----------------|-----------------|
| 0.25  | Noise Below sat. | 0.048475         | 20.9356         |
| 0.28  | Noise Below sat. | 0.041880         | 44.7202         |
| 0.30  | Noise Below sat. | 0.045057         | 51.3371         |
| 0.32  | Noise Below sat. | 0.047126         | 44.6895         |
| 0.35  | Noise Below sat. | 0.054596         | 57.8424         |
| 0.25  | 3250 Hz. 2.35Vrms | 0.042515         | 78.3571         |
| 0.25  | 3920 Hz. 4.06Vrms | 0.044337         | 99.6471         |
| 0.28  | 3250 Hz. 2.35Vrms | 0.039993         | 78.5551         |
| 0.28  | 3920 Hz. 4.06Vrms | 0.047418         | 72.5547         |
| 0.30  | 3250 Hz. 2.35Vrms | 0.051279         | 69.4989         |
| 0.30  | 3920 Hz. 4.06Vrms | 0.043100         | 39.4168         |
| 0.32  | 3250 Hz. 2.35Vrms | 0.056008         | 65.0521         |
| 0.32  | 3920 Hz. 4.06Vrms | 0.054018         | 82.9277         |
| 0.35  | 3250 Hz. 2.35Vrms | 0.058321         | 151.9084        |
| 0.35  | 3920 Hz. 4.06Vrms | 0.064033         | 196.5640        |

IV. Conclusions

This paper focuses on the modeling issues for subsonic cavity flows. An experimental setup has been constructed for this purpose and the goal is to show that the surface pressure readings could lead to a SVM based model for predicting the behavior at the cavity floor. The results have demonstrated that the goal is attainable with a simple SVM structure admitting the Mach number as one of the input variables. This makes it possible to utilize the SVM over a range of regimes characterized by the Mach number. The results obtained through the conducted research advances the subject area to the development of models based on statistical learning theory which can effectively describe the flow dynamics. The very limited number of training samples and the accuracy in extracting the features deserve emphasis. Future research aims to improve the training time of SVM based models to incorporate more input-output pairs into the regression problem.

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