Flow-acoustic resonances leading to high unsteady pressure levels may occur in flow past cavities. The long-standing semi-empirical model of Rossiter, and a more complete theoretical model recently developed by the authors, both predict the existence of several resonance frequencies. The unsteady pressure spectra measured in experiments typically also contain several resonance peaks, consistent in nature with the theory. However, in wind-tunnel experiments where the cavity is embedded in one of the wind-tunnel walls, the pressure spectrum may shift to the case of a single dominant frequency, sometimes quite abruptly and only for a narrow range of flow speeds. In the present paper, we develop a modified theoretical prediction method that explicitly accounts for the presence of wind-tunnel walls. The cross-stream eigenmodes play a central role in the theory. We show that, in the frequency (or Mach number) band where a higher-order eigenmode is cut-on in the tunnel-cavity section, but cut-off in the upstream and downstream tunnel sections, the nearly-trapped nature of the acoustic field causes a dramatic increase in the growth rate for the global flow-acoustic resonance mode. This provides an explanation for the dominant mode behavior that has been observed in experiments.

I. Introduction

Flow-acoustic resonances may occur in flow past cavities, leading to very high unsteady pressure levels. These are of concern in a variety of applications, including military aircraft weapons bays. The present paper addresses relatively shallow cavities that are representative of weapons bays. For shallow cavities, wave propagation along the length of the cavity (parallel to the flow direction) is a crucial feature of the resonance phenomenon.

The cavity acoustic resonance contains four main elements: (1) an instability wave in the free shear layer which amplifies as it propagates downstream across the face of the cavity, (2) an acoustic field that is generated by the impingement of the instability wave on the downstream edge of the cavity, (3) propagation of this acoustic field back upstream, and (4) the regeneration of the instability wave by the impingement of the acoustic field on the upstream edge of the cavity. (Other elements make secondary contributions to the resonance, as discussed later.) The resonant frequencies are determined by a phase relationship around the feedback loop that leads to constructive reinforcement. The resultant amplitude of the resonant field is controlled by a subsequent nonlinear saturation, often associated with nonlinear effects in the development of the shear-layer instability wave as it propagates across the face of the cavity.

A seminal contribution to the understanding and prediction of cavity resonances was made by Rossiter (1964), who performed an extensive experimental study of cavity resonances for free-stream Mach numbers in the range $0.5 \leq M \leq 1.2$, and also proposed a semi-empirical formula for the resonant frequencies that remains widely used even today. Rossiter's formula is based on the four-element description of the resonant loop presented in the preceding paragraph. Rossiter introduced an empirical constant to account for the phase shifts associated with the scattering processes at the two ends of the cavity (elements 2 and 4), the constant being chosen to best fit the experimental data. Heller & Bliss (1974) presented experimental results over an extended Mach number range $0.3 \leq M \leq 3.0$, and proposed a slight modification to Rossiter's formula which has little effect for the subsonic Mach numbers of interest here.
Tam & Block (1978) carried out an extensive experimental investigation at low Mach numbers. They also developed a theoretical model which involved excitation of the shear layer instability wave by the acoustic field generated at the downstream end of the cavity. The scattering process at the downstream end was represented in a simplified fashion, by introducing a concentrated acoustic monopole source at the downstream corner of the cavity. The phase of the source relative to the local instability wave motion was chosen heuristically. The excitation of the shear layer instability wave was then calculated through a \( \text{distributed receptivity} \) model, which ignored details of the interaction at the upstream end of the cavity. The amplitude of the monopole source was not determined. Therefore, the theory produced an expression for the resonance frequencies, but did not determine whether a particular resonant mode corresponded to a globally stable or unstable disturbance.

Our recent work provides a major advance in the theoretical prediction of cavity acoustic resonances. Kerschen & Tumin (2003) considered the supersonic flow case, while Alvarez, Kerschen & Tumin (2004) considered subsonic flow. In contrast to previous theories, which treated the scattering processes heuristically, our model includes a prediction for the complex scattering coefficients. This allows a full analysis of the feedback loop. Our analysis determines both the frequency and the temporal growth/decay rate of each \textquoteleft Rossiter mode'. (Previous models provided no information on growth/decay rates.)

The theoretical models presented by Kerschen & Tumin (2003) and Alvarez \textit{et al.} (2004) were for the unconfined case of a cavity under an unbounded stream. However, most experiments are carried out in wind tunnels. An important feature found in these wind tunnel experiments is the presence of a dominant mode at certain Mach numbers which are facility dependent. The dominant mode is a strong narrowband tone that may be 10 to 30 Db higher than the rest of the spectrum.

In the experiments performed by Williams, Fabris & Morrow (2000) and Ukeiley \textit{et al.} (2003), acoustic treatment was applied to the opposite cavity in the wind tunnel in an attempt to reduce effects of the adjacent tunnel walls. Both experiments, however, showed the existence of a dominant mode at some conditions. DeBiasi \textit{et al.} (2004) performed experiments for a cavity in a wind tunnel for Mach numbers \( 0.25 \leq M \leq 0.5 \). They did not introduce acoustic treatment within the wind tunnel. In their experiments, dominant mode behavior is observed over much of the Mach number range. One of the objectives of the present paper is to develop a theoretical model that explains the dominant mode behavior.

A second important issue related to dominant mode behavior was raised by Rowley \textit{et al.} (2002). They found dominant mode behavior for \( M = 0.34 \). A Mach number sweep showed no dominant mode behavior at other Mach numbers in the range \( 0.1 \leq M \leq 0.45 \). By examining the probability density function (PDF) and phase portrait, they showed that the dominant mode response at \( M = 0.34 \) had a limit cycle behavior, as expected for the nonlinear saturation of a linearly unstable mode. However, at all other Mach numbers, the PDF and the phase portrait showed that the resonance was the forced response of a stable system. This is a remarkable result which has important implications for the understanding and prediction of cavity resonances.

Our recent theory for the unconfined case (Alvarez \textit{et al.} 2004) determines growth/decay rates, so that we can examine the issue raised by Rowley. Our results show that, at low Mach numbers, the higher Rossiter modes (i.e. our global modes) are indeed stable. Hence, the presence of these modes in an experiment would correspond to the forced response of a damped system. A likely forcing mechanism is the interaction of the boundary layer turbulence with the upstream lip of the cavity. Our theoretical model for the unconfined case does not exhibit a change from stable to unstable behavior in the vicinity of \( M = 0.34 \) for the cavity geometry of Rowley.

These observations lead us to believe that the confining influence of wind-tunnel walls can have a significant influence on cavity resonances for subsonic flow. In the present paper, we develop a theory that explicitly accounts for this effect. The higher-order (cross-stream) eigenfunctions play an important role in the theory. There are two sets of such modes. The first set, which we call tunnel (T) modes, applies in the wind tunnel regions upstream and downstream of the cavity. The second set, which we call tunnel-cavity (TC) modes, applies in the region between the leading and trailing edges of the cavity. These are called tunnel-cavity modes because the cross-stream geometry for the acoustic field in this region involves both the tunnel \((0 < y' < h')\) and the cavity \((-d' < y' < 0)\). Since the cross-stream width in the tunnel-cavity region is larger than that in the tunnel region, the cut-on frequency for a higher-order TC mode is slightly lower than the cut-on frequency for the T mode of the same order. We will show that the frequency window between the cut-on frequencies of a TC mode and the corresponding T mode is particularly significant.

As a point of clarification, we note that the term \textquoteleft mode\textquoteright is used in two different contexts here. First we...
have the global modes that describe the resonant response of the complete flow-acoustic system (cavity, shear layer and wind tunnel). The ‘Rossiter’ modes for the unconfined case and the global modes we calculate for the confined case in this paper, are both global modes. Second, the field in certain regions may be described by superpositions of local eigenfunctions or modes. The tunnel-cavity and tunnel modes are local eigenfunctions, as is the shear layer instability mode.

Koch (2004) investigated cavity acoustic resonances in unconfined and confined settings, for the case of zero mean flow. Since mean flow effects were not included, his theory does not incorporate the energy input from shear layer instabilities that plays a central role in our theory. However, his work clearly identifies the important differences between the acoustic response of cavities in unconfined and confined settings. An unconfined cavity loses energy by acoustic radiation; therefore the resonant frequencies of the global modes are complex, with negative imaginary parts corresponding to temporal damping of the global mode. A cavity in the wall of a wind tunnel has a different acoustic environment, which modifies the complex resonant frequencies of the global modes, changing both the temporal frequencies and the damping rates. For the confined setting, acoustic radiation takes place through the cut-on tunnel modes. If the global resonance does not involve the excitation of any cut-on tunnel modes, there is no acoustic radiation and the energy of the mode is ‘fully trapped’. Koch (2004) showed that fully trapped global modes occur for a cavity in the wall of a wind tunnel. For a fully trapped global mode, the imaginary part of the resonant frequency is identically zero and the global mode has no damping.

The flow-acoustic resonances that we analyze in this paper have some similarities with Koch’s fully trapped global modes. For a cavity in the wall of a wind tunnel with mean flow, the symmetry conditions required for Koch’s fully trapped global modes are generally not realized. However, a nearly trapped global mode, which has only relatively weak acoustic radiation, can occur in the frequency window between the cut-on frequency of a tunnel-cavity mode and the cut-on frequency of the corresponding tunnel mode. The cut-on TC mode typically has a large amplitude, while the corresponding T mode is cut-off and cannot radiate energy upstream or downstream. Hence, the acoustic field in the TC region is nearly trapped, with very little radiation damping. This leads to growth rates for the global flow-acoustic resonance that are much larger than those outside this frequency band. In fact, this mechanism can cause a fundamental change in the character of the flow-acoustic resonance, from a damped global mode to an unstable global mode. We shall show that this mechanism provides an explanation for the dominant mode behavior that has been observed in various experiments.

II. Analysis

Consider a two-dimensional wind tunnel with a height $h'$ and a subsonic speed $U$. A rectangular cavity of length $L'$ and depth $d'$ is mounted in the lower wall. The origin of the coordinate system is placed on the lower wall of the wind tunnel at the upstream edge of the cavity. The total field is the sum of a mean field and a time-harmonic perturbation field with time dependence $\exp(-i\omega t)$. The complex frequency is $\omega_c = \omega(1 + i\eta)$, where $\omega$ and $\eta$ are real.

We define non-dimensional variables

$$ (x, y) = \frac{\omega}{U} (x', y'). $$

The non-dimensional forms of the tunnel height and the cavity depth and length are then

$$ h = \omega h'/U, \quad d = \omega d'/U, \quad L = \omega L'/U. $$

The global model for the unsteady field in the cavity is described in subsection A. The local analyses for the scattering problems at the two ends of the cavity are described in subsection B, and the analyses for the propagation of disturbances across the central portion of the cavity are described in subsection C.

A. The global model

For reference, we first describe the global model for our previous theory, which considered a cavity under an unbounded stream. In this case the resonance has the following elements: a shear-layer instability wave ($S$), upstream- and downstream-propagating acoustic modes in the cavity ($U$ and $D$), and upstream- and downstream-propagating external acoustic fields ($E_u$ and $E_d$), as illustrated in Fig. 1.

For a cavity in the wall of a wind tunnel, as illustrated in Fig. 2, the situation is somewhat different. We consider two types of regions: the tunnel regions upstream and downstream of the cavity ($x < 0$ and $x > L$),
Figure 1. Illustration of the global model for cavity acoustic resonances, for the case of an unbounded subsonic stream.

and the tunnel-cavity region \((0 < x < L)\). The global model in the wind tunnel case contains the following elements. Within the tunnel-cavity region, we have the shear-layer instability wave \((S)\) and upstream- and downstream-propagating acoustic modes \((U_0', U_1', U_2', \ldots, D_0', D_1', D_2', \ldots)\) which we call TC modes. For the parameter range we consider, the modes \(U_j\) and \(D_j\) for \(j = 2, 3, \ldots\), are cut-off and can be neglected. We also have additional upstream- and downstream-propagating acoustic modes that are concentrated in the cavity region \((y < 0)\). These modes exhibit strong exponential decay in the propagation direction and will not be included in our global model. Finally we have upstream-propagating tunnel modes \((T_0^u, T_1^u, T_2^u, \ldots)\) in the region \(x < 0\), and downstream-propagating tunnel modes \((T_0^d, T_1^d, T_2^d, \ldots)\) in the region \(x > L\).

The various elements in the tunnel-cavity region are coupled by the scattering processes at \(x = 0\) and \(x = L\). First consider the scattering process at the upstream end of the tunnel-cavity region \((x = 0)\). The local amplitudes of all quantities at the upstream end \((x = 0)\) are denoted by the decoration \((\hat{\cdot})\). The scattering process at the upstream end involves the impingement of the upstream-propagating TC modes of amplitude \(\hat{U}_0\) and \(\hat{U}_1\), which produce three downstream-propagating components. The (complex) amplitudes of the downstream-propagating components generated by this scattering process are given by

\[
\begin{bmatrix}
\hat{S} \\
\hat{D}_0 \\
\hat{D}_1
\end{bmatrix} =
\begin{bmatrix}
C_{S\hat{U}_0} & C_{S\hat{U}_1} \\
C_{D_0\hat{U}_0} & C_{D_0\hat{U}_1} \\
C_{D_1\hat{U}_0} & C_{D_1\hat{U}_1}
\end{bmatrix}
\begin{bmatrix}
\hat{U}_0 \\
\hat{U}_1
\end{bmatrix},
\] (3)

where the six scattering coefficients for the upstream end, \(C_{S\hat{U}_0}, C_{D_0\hat{U}_0}, C_{D_0\hat{U}_1}, C_{S\hat{U}_1}, C_{D_1\hat{U}_0}\) and \(C_{D_1\hat{U}_1}\), are determined by the procedure discussed in subsection B.

Next consider the scattering process at the downstream end of the tunnel-cavity region. Quantities without decoration denote the local amplitudes at the downstream end \((x = L)\). The scattering process at the downstream end involves the impingement of the three downstream-propagating components, each of which makes a contribution to the amplitude of the upstream-propagating TC modes \(U_0\) and \(U_1\),

\[
\begin{bmatrix}
U_0 \\
U_1
\end{bmatrix} =
\begin{bmatrix}
C_{U_0S} & C_{U_0D_0} & C_{U_0D_1} \\
C_{U_1S} & C_{U_1D_0} & C_{U_1D_1}
\end{bmatrix}
\begin{bmatrix}
S \\
D_0 \\
D_1
\end{bmatrix}.
\] (4)

The six scattering coefficients for the downstream end, \(C_{U_0S}, C_{U_0D_0}, C_{U_0D_1}, C_{U_1S}, C_{U_1D_0}\) and \(C_{U_1D_1}\), are determined by the procedure discussed in subsection B.

Figure 2. Illustration of the global model for cavity acoustic resonances for subsonic flow in a wind tunnel.
Finally, consider the evolution of all components as they propagate from one end of the tunnel-cavity region to the other. For the sake of convenience in formulating the analysis, we consider the case where the propagation characteristics are not a function of $x$. However, the streamwise dependence of the propagation characteristics is accounted for in the actual calculations, as discussed in subsection C. For the downstream-propagating components, the changes in the complex amplitudes of the components in propagating from the upstream end to the downstream end are given by

$$ S = S e^{i \alpha L}, \quad D_0 = D_0 e^{i \delta D_0 L}, \quad D_1 = D_1 e^{i \delta D_1 L}. \quad (5) $$

Here, $\alpha$, $\delta D_0$, and $\delta D_1$ are the wavenumbers of the instability wave and the downstream-propagating TC modes $D_0$ and $D_1$, respectively.

Similarly, for the upstream-propagating components, the changes in the complex amplitudes in propagating from the downstream end to the upstream end are given by

$$ \hat{U}_0 = U_0 e^{i \delta U_0 L}, \quad \hat{U}_1 = U_1 e^{i \delta U_1 L}. \quad (6) $$

Here $\delta U_0$ and $\delta U_1$ are the wavenumbers of the upstream-propagating TC modes $U_0$ and $U_1$, respectively.

The global eigenrelation is obtained by combining the above equations. Consider the unknown amplitude vector

$$ X = \begin{bmatrix} U_0 \\ U_1 \\ S \\ D_0 \\ D_1 \end{bmatrix}. \quad (7) $$

Substituting (5) into (4), and (6) into (3), we can form the matrix equation $AX = 0$, where

$$ A = \begin{bmatrix} -1 & 0 & C_{U_0 S} e^{i \alpha L} & C_{U_0 D_0} e^{i \delta D_0 L} & C_{U_1 D_1} e^{i \delta D_1 L} \\ 0 & -1 & C_{U_0 S} e^{i \alpha L} & C_{U_1 D_0} e^{i \delta D_0 L} & C_{U_1 D_1} e^{i \delta D_1 L} \\ C_{S D_0} e^{i \delta U_0 L} & C_{S D_1} e^{i \delta U_1 L} & -1 & 0 & 0 \\ C_{D_0 D_0} e^{i \delta D_0 L} & C_{D_0 D_1} e^{i \delta D_1 L} & 0 & -1 & 0 \\ C_{D_1 D_0} e^{i \delta U_0 L} & C_{D_1 D_1} e^{i \delta U_1 L} & 0 & 0 & -1 \end{bmatrix}. \quad (8) $$

Calculating the determinant, we obtain the eigenvalue relation

$$ e^{i L (\alpha + \delta U_0)} C_{S D_0} C_{U_0} S + e^{i L (\delta D_0 + \delta U_1)} C_{D_0 D_0} C_{U_0} D_0 + e^{i L (\alpha + \delta U_1)} C_{S D_1} C_{U_1} S + e^{i L (\delta D_0 + \alpha)} C_{D_0 D_1} C_{U_0} D_1 + e^{i L (\delta D_1 + \delta U_0)} C_{D_0 D_1} C_{U_1} D_0 + e^{i L (\delta D_1 + \delta U_1)} C_{D_1 D_0} C_{U_0} D_0 + e^{i L (\delta D_1 + \delta U_0)} C_{D_1 D_1} C_{U_1} D_0 + e^{i L (\delta D_1 + \delta U_1)} C_{D_1 D_1} C_{U_1} D_1 \quad (9) $$

The complex frequency enters implicitly in the propagation wavenumbers $(\delta U_0, \delta U_1, \alpha, \delta D_0$, and $\delta D_1)$ for the components of the disturbance field $(U_0, U_1, S, D_0$ and $D_1)$, and in the various scattering coefficients ($C_{S D_0}$, etc.). This eigenvalue relation contains all possible feedback paths involving the two upstream-propagating components $(U_0$ and $U_1$) and the three downstream-propagating components $(S, D_0$ and $D_1)$. Our theoretical model incorporates non-parallel mean flow effects, so that the simple propagation factors in (9) are replaced by integrals along the length of the cavity.

In our previous work for cavity resonance under an unbounded subsonic stream (Fig. 1), we found that only $S$ and $U$ were important, so that $D$, $E_u$, and $E_x$ could be neglected. This allowed the eigenvalue relationship to be simplified dramatically, to the case of a single loop. The final result (see Alvarez et al. 2004) was a prediction for the resonant frequency that had much in common with the Rossiter formula, but contained no empirical constants. The predictions from our theoretical model were compared with
experimental data, and produced significantly better agreement with the experimental data than the Rossiter prediction. This is an impressive result, when one considers that Rossiter’s formula has empirical constants that can be adjusted to fit the data, while our model has no such constants.

For the case involving wind-tunnel walls, we again find that many of the loops in the full eigenvalue relationship (9) can be neglected. However, for the specific case we have examined in detail \((L/d = 5, h/d = 4\) and \(M = 0.35\)), we find that two loops must be included in order to calculate the lowest resonant frequency, and three loops must be included in order to capture the dominant mode behavior.

In the following subsections, we discuss the calculation of the various scattering and propagation coefficients which enter in the eigenvalue relationship for the global modes.

B. Calculation of scattering coefficients for the upstream and downstream ends

The local analyses for the scattering processes at the ends of the tunnel-cavity region consider a finite-length overhanging lip geometry, as illustrated for the upstream end in Fig. 3. In these local analyses, the shear layer is approximated by a vortex sheet.

For the scattering process at the upstream end, the incident fields are the upstream-propagating TC modes \(U_0\) and \(U_1\). For the scattering process at the downstream end, the incident field is the shear-layer instability wave \(S\) and the downstream-propagating TC modes \(D_0\) and \(D_1\). The TC modes are acoustic fields that satisfies the no-penetration boundary condition on the cavity bottom and on the tunnel top, and matching conditions of continuity of pressure and particle displacement across the vortex sheet. The lowest-order TC modes, \(U_0\) and \(D_0\), propagate without attenuation at all frequencies.

The analysis for the scattering process at each end involves the following steps. First, the end wall of the cavity is removed and the case of a semi-infinite overhanging lip is considered. The geometry is then amenable to application of the Wiener–Hopf technique (Noble 1988). The scattered field produced by impingement of the incident field consists of an infinite set of TC modes that are reflected into the cavity region, an infinite set of acoustic duct modes that propagate into the ‘duct’ formed by the cavity bottom and the overhanging lip, and an infinite set of tunnel modes that propagate into the tunnel. For the scattering process at the upstream end, a shear-layer instability wave is also generated.

A second scattering problem for the case of a semi-infinite overhanging lip is then solved, in which the incident field is an acoustic duct mode propagating toward the cavity. This also generates a scattered field containing the same components as described in the preceding paragraph.

The solution for the finite-length overhanging lip geometry, including the presence of the end wall, is then obtained by noting that the complete field under the overhanging lip is composed of an infinite set of upstream- and downstream-propagating duct modes. Applying the no-penetration condition on the cavity end wall, an infinite-dimensional matrix equation for the duct mode coefficients is obtained. However, for a fixed frequency, only a finite number of the duct modes are cut-on. (In many cases, only a single duct mode is cut-on.) The rest decay exponentially away from their point of origin and have insignificant amplitudes at the location of the end wall. The infinite-dimensional matrix equation can then be truncated to finite order.

Figure 3. Illustration of the finite-length overhanging lip geometry for the upstream end of the cavity. The geometry for the downstream end of the cavity is similar.
and solved numerically. The rapid convergence of the solutions with increasing matrix order is remarkable. In fact, the procedure works well even when the overhang length $b$ is set to zero, and the exponential decay of the higher-order terms is replaced by algebraic decay.

Once the amplitudes of the duct modes under the finite-length overhanging lip are determined, the coefficients of the scattered field components that propagate into the cavity region are fully determined. These coefficients include the scattering coefficients introduced in subsection A. The scattered field in the cavity region consists of an infinite number of TC modes, and a shear-layer instability wave in the case of the scattering process at the upstream end. However, for the frequencies of interest, the higher-order TC modes ($j = 2, 3, \ldots$) are cut-off and decay exponentially away from their point of origin. Hence, the unsteady field in the central portion of the cavity can be approximated by the finite number of elements considered in subsection A, connected by the scattering coefficients determined by the local analyses for the upstream and downstream ends.

C. Calculation of the propagation wavenumbers

The determination of cavity resonance frequencies requires accurate predictions of the phase evolution of the instability wave and the TC modes as they propagate the length of the cavity, while accurate prediction of the resonance temporal growth (or decay) rate requires accurate predictions of the amplitude evolution of these disturbances as they propagate the length of the cavity. We first discuss calculation of the instability wave propagation, and then discuss calculation of the TC mode propagation.

In order to accurately predict the evolution of the instability wave, we must account for both the non-parallel effects due to the streamwise development of the shear layer, and the influence of fine-grain turbulence on the instability wave. The instability wave evolution is calculated using the method of Yang & Tumin (2002). Their method utilizes the triple decomposition technique of Reynolds & Hussain (1972). In this technique, the flow field is decomposed into three components: a mean (time-averaged) flow, a coherent (phase-averaged) component, and random (turbulent) motion. The coherent component is the instability wave of interest for the cavity resonance. The influence of the random turbulent fluctuations on the large-scale coherent disturbance is approximated by the Newtonian eddy viscosity model. The governing equations for the coherent disturbances have the same form as in laminar flow, with substitution of the Reynolds number and the Prandtl number by their turbulent counterparts.

The solution for the (coherent) instability wave component is developed using the method of multiple scales. At leading order, an eigenvalue problem is found for the local value of the instability wavenumber at each $x$-station. The streamwise variation of the wavenumber is significant, due to the streamwise development of the mean shear layer. The amplitude equation which arises at second order accounts for non-parallel mean flow effects.

The propagation of the TC modes can be analyzed very similarly. The governing equations are basically the same as those used for the instability wave development, except for changes in the boundary conditions. Again, the triple-decomposition method is applicable. However, in contrast to the instability wave, the acoustic tunnel-cavity modes are not concentrated in the shear layer, but rather extend across the full height of the tunnel. Therefore, the propagation of the TC modes is influenced less by details of the shear layer. Thus, for the predictions presented here, the vortex-sheet approximation has been used to calculate the propagation of the TC modes.

III. Results

Prediction of the cavity flow-acoustic resonances requires specification of the geometry, the flow field (including details of the shear layer), and the acoustic boundary conditions. In the experiments of Williams et al. (2000) and Ukeiley et al. (2003), the tunnel wall opposite the cavity was treated with acoustically absorbing material whose details were not specified. In the experiments of DeBiasi et al. (2004), all interior surfaces of the tunnel were rigid, but details of the shear layer were not specified. In the absence of further information, it is not possible to make precise comparisons with any of these experiments.

Therefore, to illustrate the significant effects of wind-tunnel walls, we consider the following typical case. We choose a cavity with $L/d = 5$, $M = 0.35$ and initial shear layer properties matching those of Williams et al. (2000). Yang and Tumin (2002) presented a detailed analysis of the mean flow and instability properties of the shear layer for this case, and validated features of their predictions against the experimental data.
Hence, we can use the predictions of Yang and Tumin with confidence. For the wind-tunnel geometry we choose $h' = 50.8$ mm and $h/d = 4$, matching the geometry of Debiasi et al. Dimensional results that are presented below have been scaled to match the values of $h'$ and $d'$ in Debiasi’s experiments. Note, however, that Debiasi’s cavity length was $L/d = 4$, so our predictions should not be compared directly to their experimental results.

For $M = 0.35$ and $h' = 50.8$ mm, the cut-on frequencies of the $j = 1$ tunnel-cavity modes and tunnel modes are $f = 2555$ Hz and $3162$ Hz, respectively. The corresponding non-dimensional frequencies are $d = 1.70$ and $2.10$, respectively.

The importance of specific terms in the full eigenvalue relationship (9) depends on both the propagation factors and the scattering coefficients. First consider the propagation factors. The only propagation factor that contains exponential growth is that for the shear layer instability wave $S$. For the case considered here, the factor $|\exp(i\alpha L)|$ takes on a maximum value of 11.1 at $f = 1585$ Hz. Thus, terms in (9) that contain $\exp(i\alpha L)$ are likely to be important. The lowest-order TC modes ($U_0$ and $D_0$) have wavenumbers ($\delta_{D_0}$ and $\delta_{U_0}$) that are purely real for all frequencies, so these modes propagate without attenuation. The $j = 1$ TC modes also propagate without attenuation for frequencies above their cut-on frequency, while below this frequency they suffer exponential attenuation at a rate that increases as the frequency is reduced.

Next consider the magnitudes of the scattering coefficients. The six one-pass loops in the eigenvalue relationship (9) contain products of two scattering coefficients, while the two-pass loops contain products of four scattering coefficients. Since the scattering coefficients are often smaller than one, the one-pass loops are typically more important than the two-pass loops. Maginitudes of the scattering coefficients for the two one-pass loops that contain $\exp(i\alpha L)$ are plotted in Fig. 4. The magnitudes of the scattering coefficients for $S-U_0$ loop are plotted in Fig. 4(a). The magnitudes of the scattering coefficients for $S-U_0$ loop are fairly strong inside the frequency window, but the scattering from this mode into the $j = 0$ mode is relatively weak. Thus, these loops are less important than the $D_1-U_1$ loop. The two-pass loops involve products of four scattering coefficients, of which one or more is typically small; therefore the two-pass loops are generally less important.

Based on the above discussion, we determined that accurate predictions for the present case could be

![Figure 4](image-url)

Figure 4. Magnitudes of the scattering coefficients for the one-pass loops that contain the instability wave, plotted as a function of frequency $d = \omega d'/U$ for $M = 0.35$ and $h/d = 4$. (a) $|C_{U_0 S}|$, solid; $|C_{S U_0}|$, dashed. (b) $|C_{U_1 S}|$, solid; $|C_{S U_1}|$, dashed.
At low frequencies, the magnitude and phase for the three individual loops are plotted as a function of (real) frequency. In Fig. 6, the magnitude and phase for the combined three-loop eigenvalue relationship (i.e. the left-hand side of (10)) are plotted for real values of the frequency. In general, the global-mode eigenvalue relationship is

$$ e^{jL(\alpha + \delta U_0)}C_{C\ell_0}C_{U_0} + e^{jL(\alpha + \delta U_1)}C_{C\ell_1}C_{U_1} + e^{jL(\delta D_1 + \delta U_1)}C_{D_1U_1C_{U_1}D_1} = 1 $$

(10)

In Fig. 6, the magnitude and phase for the three individual loops are plotted as a function of (real) frequency. At low frequencies, the $S-U_0$ loop is most important, but the $S-U_1$ loop makes a secondary contribution. The importance of the latter loop arises because the scattering coefficients are quite large, partially negating the exponential attenuation of the $U_1$ eigenmode at frequencies below its cut-on frequency. Below the $j = 1$ cut-on frequency ($f = 2555$ Hz), the $D_1-U_1$ loop suffers double exponential attenuation and its magnitude is negligible. However, in the frequency window between the cut-on frequencies of the $j = 1$ tunnel-cavity and tunnel eigenmodes, the scattering coefficients for the $D_1-U_1$ loop are very large and this loop becomes the largest contribution to the simplified eigenvalue relationship (10). Above the cut-on frequency for the $j = 1$ tunnel mode ($f = 3162$ Hz), the scattering coefficients for the $D_1-U_1$ loop decrease quite rapidly.

The arguments for the three individual loops are also shown in Fig. 6. The arguments exhibit smooth variations with frequency at frequencies below the cut-on frequency for the $j = 1$ tunnel-cavity eigenmodes. The arguments of the $S-U_0$ and $S-U_1$ loops are similar in this region. The argument of the $D_1-U_1$ loop exhibits rapid variations in the vicinity of the cut-on frequencies for the $j = 1$ tunnel-cavity modes and tunnel modes. Some rapid variations in the arguments of the $S-U_0$ and $S-U_1$ loops are also seen near the cut-on frequencies. The rapid variations in the arguments of the loops is caused by rapid variations in the arguments of the scattering coefficients in the vicinity of the cut-on frequencies.

In Fig. 7, the magnitude and argument for the combined three-loop eigenvalue relationship (i.e. the left-hand side of (10)) are plotted for real values of the frequency. In general, the global-mode eigenvalue relationship corresponds to a complex frequency $\omega_c = \omega' + injL'/U$. However, results for real frequency can be used to obtain an approximate value for $\omega$, and to determine whether $\eta$ is positive or negative (unstable or stable global mode). The resonance frequencies correspond to values of the argument equal to $m\pi$. The global mode is unstable if the magnitude at the corresponding real frequency is greater than one, while the global mode is stable if the magnitude is less than one.

From Fig. 7(b), the frequencies of the global modes for the wind-tunnel case are seen to be approximately $f = 1100, 2050, 2817, 3801$ Hz. In addition, the rapid variation of the phase near the cut-on frequency of the TC mode suggests that additional global modes are present at approximately $f = 2515$ and $2555$ Hz. (Because these two frequencies are so close together, they could merge into a single frequency when the computation is extended into the complex frequency plane.) From Fig. 7(a), we see that the amplitudes at $f = 1100, 2050, 2525$ and $3801$ Hz are much less than one, so these global modes are damped (stable). In contrast, the amplitude at $f = 2555$ Hz is greater than one, while that at $2817$ Hz is very close to one. Thus, the former is clearly an unstable global mode, while the latter is on the borderline between stable and unstable behavior. Since the amplitudes for these two frequencies are much higher than those for the other global modes, our theory implies the presence of a dominant mode response in this frequency range.
Figure 6. Magnitudes and arguments of the three loops in the simplified eigenvalue relationship (10), as a function of frequency. (a) $S-U_0$ loop, (b) $S-U_1$ loop, (c) $D_1-U_1$ loop.
For comparison, the magnitude and argument of the simplified eigenvalue relationship for the unconfined case (Alvarez et al. 2004) are plotted as a function of frequency in Fig. 8. From the argument plot, the resonance frequencies for the unconfined case are seen to be approximately \( f = 1260, 2180 \) and 32340 Hz. From the magnitude plot, the mode at \( f = 1260 \) Hz is seen to be unstable, while the other modes are stable.

Comparing the results for the confined and unconfined cases, we see that the wind-tunnel walls have caused some shifts in the frequencies of the global modes, and also influenced the growth/decay rates. It is interesting that the first global mode is unstable in the unconfined case, but stable in the confined case. Most importantly, the wind-tunnel walls have caused the global modes in the frequency window, between the cut-on frequencies of the \( j = 1 \) tunnel-cavity and tunnel modes, to become unstable. Since all other modes are strongly damped in the wind-tunnel case, one would expect a dominant mode response in the frequency window.

The results presented above consider the frequency response at a fixed Mach number. Cavity resonance experiments often examine the behavior as a function of Mach number. In order to illustrate the dominant mode behavior as a function of Mach number that would be predicted by our theory, consider the results plotted in Fig. 9. The geometry is \( h/d = 4 \) and \( L/d = 5 \), with \( h' = 50.8 \) mm. The solid lines in this plot are the estimated cavity resonance frequencies, from the Rossiter formula. The first five Rossiter modes (in order of increasing frequency) are shown on the plot. The lowest dashed line is the cut-on frequency for the \( j = 1 \) tunnel-cavity mode, while the dashed line slightly above this is the cut-on frequency for the \( j = 1 \) tunnel mode. Dominant mode response is likely to occur in the frequency window between the two dashed lines. Similarly, the upper set of dashed lines are the cut-on frequencies for the \( j = 2 \) tunnel-cavity and tunnel modes, respectively. The frequency window between these two dashed lines is also a likely region for dominant mode response.

For the case of considered in Fig. 9, the third Rossiter mode enters the frequency window at \( M = 0.29 \)
Figure 9. Illustration of the frequency windows where dominant mode resonance is likely to occur, as a function of Mach number. Rossiter modes 1–5 are plotted as solid lines (lower to upper). Frequency windows lie in the narrow bands between the dashed lines ($j = 1$ window at 3000 Hz, $j = 2$ window at 6000 Hz).

and leaves at $M = 0.36$. The second Rossiter mode then enters the frequency window at $M = 0.46$ and leaves at $M = 0.56$. While the actual resonant frequencies in the wind-tunnel setting will be somewhat different than the Rossiter predictions (as illustrated by comparison of Figs. 7(b) and 8(b)), the results in Fig. 9 identify the approximate regions in Mach number (and frequency) space where dominant mode response is likely to occur.

The conditions considered in Fig. 9 are reasonably close to the experiments of DeBiasi et al. (2004), which considered $L/d = 4$ and $h/d = 4$ with $h' = 50.8$ mm. Figure 2 of DeBiasi et al. is a plot of resonant mode frequencies as a function of Mach number, similar to our Fig. 9. The dominant mode behavior in these experiments follows closely the trend illustrated in Fig. 9. The third Rossiter mode lies in the frequency window and is dominant at low Mach numbers, while the second Rossiter mode lies in the frequency window and is dominant at higher Mach numbers. (Since DeBiasi’s geometry has both $L/d$ and $h/d$ equal to 4, the 1st transversal and 1st longitudinal mode frequencies shown on their plot are close to the lower and upper bounds of the $j = 1$ frequency window of our theory.) Figure 2 of DeBiasi et al. also shows that the wind-tunnel environment produces some shifts of the resonant frequencies away from the predictions of the Rossiter formula. Finally, the resonant frequencies in the experiments are seen to undergo rapid changes near the lower and upper bounds of the frequency window. This behavior is also consistent with our theory, since the arguments of the scattering coefficients vary rapidly in the vicinity of the cut-on frequencies for the tunnel-cavity and tunnel modes.

IV. Conclusions

We have developed a theoretical model for cavity flow-acoustic resonances that incorporates the effects of wind-tunnel walls. The theory shows that wind-tunnel effects can dramatically change the flow-acoustic response in frequency windows where a cross-stream mode in the tunnel-cavity region is cut-on, but the corresponding cross-stream mode in the tunnel is cut-off. This leads to a ‘trapping’ of energy in the tunnel-cavity region. Although some energy still escapes through lower-order cross-stream modes, the increased energy trapping in this situation increases the linear growth rate of the global mode, so that higher amplitudes are required in order to saturate this mode through nonlinear effects. We believe this mechanism explains the dominant mode response that has been observed in some cavity resonance experiments performed in wind tunnels.

The theory has been developed assuming that all interior surfaces in the wind tunnel are rigid. However, it could be extended to incorporate acoustic treatment on the wind-tunnel walls, or other features or modifications of existing or proposed experimental configurations. The analysis of such features would allow their efficacy to be examined prior to the construction or modification of a facility.
The general ‘mode-trapping’ phenomenon that has been investigated in this paper also has applications to other flow systems with complex duct geometries. Complex duct geometries are found in a variety of applications, including military and commercial aircraft, and ground-based facilities such as power plants and engine test cells. The type of theory that has been developed here may also be useful in identifying and diagnosing resonances that occur in these other applications.

References


